Exercise 6A

1 H_0: There is no difference between the observed and expected distributions.
H_1: There is a difference between the observed and expected distributions.

2 a H_0: The observed data are drawn from a discrete uniform distribution. (The dice is fair.)
H_1: The observed data are not drawn from a discrete uniform distribution. (The dice is not fair.)

b The observed and expected results are:

Number, n	**1**	**2**	**3**	**4**	**5**	**6**
Observed (O_i)	27	33	31	28	34	27
Expected (E_i)	30	30	30	30	30	30
$\frac{(O_i - E_i)^2}{E_i}$	0.3	0.3	0.033	0.133	0.533	0.3

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 1.6$$

3 a H_0: The observed data are drawn from a discrete uniform distribution.
H_1: The observed data are not drawn from a discrete uniform distribution.

b If the distribution of students is uniform then each year group would be expected to have:
$$\frac{750}{5} = 150 \text{ students}$$

c The observed and expected results are:

Year	**7**	**8**	**9**	**10**	**11**
Observed (O_i)	190	145	145	140	130
Expected, E_i	150	150	150	150	150
$\frac{(O_i - E_i)^2}{E_i}$	10.667	0.167	0.167	0.667	2.667

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 14.33$$

4 a The observed and expected results are:

Mutation present	**Yes**	**No**
Observed (O_i)	117	43
Expected (E_i)	120	40

b H_0: The underlying probability of 'Yes' is 0.75.
H_1: The underlying probability of 'Yes' is not 0.75.

4 c $X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{3^2}{120} + \frac{3^2}{40} = 0.3$

5 a The observed and expected results are:

Result	H	T
Observed (O_i)	28	22
Expected (E_i) for fair coin	25	25
Expected (E_i) for biased coin	30	20

b For fair coin:

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{3^2}{25} + \frac{3^2}{25} = 0.72$$

For biased coin:

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{2^2}{30} + \frac{2^2}{20} = 0.33$$

c The value of X^2 is greater for the fair coin so it is more likely that John has been using the biased coin.

6 The observed and expected results are:

BMI profile	Underweight	Normal	Overweight	Obese
Observed (O_i) for men	4	70	80	46
Expected (E_i)	4	70	72	54
$\frac{(O_i - E_i)^2}{E_i}$	0	0	0.889	1.185
Observed (O_i) for women	6	81	65	48
Expected (E_i)	4	70	72	54
$\frac{(O_i - E_i)^2}{E_i}$	1	1.729	0.681	0.667

For men:

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 2.074$$

For women:

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 4.076$$

The men have a lower X^2 statistic so more closely match the English distribution.